Emergence of cooperation among interacting individuals

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We study the evolution of interacting individuals located on the sites of a regular lattice. The individuals play a two action game in which the players either cooperate or defect with respect to a certain issue. The main rule of the game is that a player does not change his action when he and his opponent have held the same action in the previous round. Numerical simulations performed on a square lattice show a stationary state in which the lattice has a finite number of cooperators and defectors and two frozen states, one full of cooperators and the other full of defectors. [S1063-651X(99)03806-4]

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I. INTRODUCTION

The problem of the conditions for emergence of cooperation among individuals who behave according to their self-interest was introduced and analyzed by Axelrod [1]. The analysis is made within the framework of game theory by the use of a *two action* game known as the prisoner's dilemma in which the players either cooperate or defect with respect to a certain issue. The prisoner's dilemma game is such that for any choice of one player, cooperation or defection, the other gets a higher payoff by choosing the defensive strategy of always playing defection. However, if both players choose such a defensive strategy the total payoff will be smaller than the total payoff they would get if both had chosen cooperation. This is the static solution of the game.

The dynamical approach to the problem was introduced by Axelrod [1] and widely used by other authors [2,3]. At each move, the players revise their choices and play according to specific rules or, using a more appropriate term used in game theory, *strategies*. The next move may depend only on the last move or may include the whole history of the game so far. The strategy may be deterministic, such as the tit-for-tat strategy, or probabilistic. Here we consider the evolution of the game to be a Markov process in which strategies are probabilistic ones and depend only on the last move. Moreover, the Markov process is considered to be of the continuous time type, described by the master equation, in contrast to a probabilistic cellular automaton in which the states are updated synchronously.

In this paper we deal with the problem of finding the conditions for the emergence of cooperation in the evolution of games within the context of the technical resources of a representation by lattice gases and its formalism. Thus, we present models for the evolution of N interacting individuals located on the sites of a lattice. The individuals play the two action game only with individuals belonging to a small neighborhood which intends to simulate real interactions among individuals of a population. Similar spatial games have been considered before by several authors [4–9]. In a narrower context, we come to focus more specifically on the work developed by Szabó and Tőke [9] and related to it we aim, in this article, to obtain similar results with distinct and much more simplified rules to be described below.

The main rule of the game is that a player does not change his action when he and his opponent have held the same action in the previous round. This leads to the existence of two frozen (absorbing) states, one in which the lattice is full of cooperators and the other full of defectors. Numerical simulations were carried out in one and two dimensions. Simulations on a square lattice have shown that besides the two frozen states there is also a stationary active state in which the lattice has a finite density of cooperators and defectors. Our simulations show that the active state is present in two dimensions but not in one dimension.

II. MODEL

Consider a population of N players located on the sites of a regular lattice. The game evolves as a succession of rounds occurring at time intervals in which the individuals play either cooperation (C) or defection (D). At each time step a player, chosen at random, revises his action according to a rule (strategy) that specifies the outcome depending upon what his neighbors have played in the previous round. The model is a stochastic process governed by a master equation whose transition rates will be given below.

A given player interacts only with the players belonging to a small neighborhood, here defined to be the nearest neighbor players. But, following Axelrod [1], we assume that a given player interacts with them one at a time. Four possible outcomes may occur in a two action game between a given player and his opponent: CC, CD, DC, and, DD, which Nowak and collaborators [3] call ''reward,'' ''loss,'' ''temptation,'' and ''punishment,'' respectively. The rules we propose for conducting the game are distinct from those advanced by Szabó and Tőke [9].

First of all, a player is chosen at random, say player i. Second, this player revises his action in the following way.

A neighboring opponent, say player j, is chosen also at random.

(a) If in the previous round both players, i and j, have held the same action, that is, if the previous outcomes were ''reward'' or ''punishment,'' then the player i does not change his action and plays as before.

(b) If, on the other hand, they have held distinct actions then two cases should be considered.

(1) Player i has played C and player j has played D, that is, the previous outcome was ''loss.'' In this case player i plays D, with a transition rate a.

(2) Player i has played D and player j has played C, that is, the outcome was ''temptation.'' In this case another neighboring opponent, say player k, is chosen at random. If player k has played C then player i changes his action, and plays C, with a transition rate b_1. If, however, player k has played D then player i changes his action, and plays C, with a transition rate b_2.

The model has three parameters a, b_1, and b_2, but they can be reduced to only two if one rescales the time. Thus we define the retaliation parameter $r=a/b_2$ and the cooperation parameter $c=b_1/b_2$. The parameter r may be interpreted as the strength with which a player retaliates against his opponent who has defected when he has cooperated. Increasing the strength of retaliation r we expect a decrease in the number of cooperators. The parameter c gives the degree of cooperation among individuals. Increasing c we expect an increase in the number of cooperators.

In a formal way, to each site i we attach a dynamic variable η_i that takes the values 0 or 1 according to whether the player at site i chooses defection or cooperation, respectively. The transition rate $w_i(\eta)$ from η_i to $1-\eta_i$ is then given by

$$w_i(\eta)=\frac{1}{z}\sum_j \left(\eta_i(1-\eta_j)a+(1-\eta_i)\eta_j\frac{1}{z-1} \times \sum_{k(\neq j)}[b_1\eta_k+b_2(1-\eta_k)]\right), \quad (1)$$

where j and k are nearest neighbors of i and z is the lattice coordination number. The master equation that governs the time evolution of the probability $P(\eta,t)$ of $\eta=(\eta_1,\eta_2,\ldots,\eta_N)$ at time t is given by

$$\frac{d}{dt}P(\eta,t)=\sum_i \{w_i(\eta^i)P(\eta^i,t)-w_i(\eta)P(\eta,t)\}, \quad (2)$$

where $\eta^i=(\eta_1,\ldots,1-\eta_i,\ldots,\eta_N)$.

We are mainly interested in the density of cooperators $\rho=\langle\eta_i\rangle$. In the simple mean field approximation the time evolution of this quantity is given by

$$\frac{d}{dt}\rho=\rho(1-\rho)[-a+b_1\rho+b_2(1-\rho)]. \quad (3)$$

The stationary solutions are $\rho=0$, the full defection (D) state, $\rho=1$, the full cooperation (C) state, and

$$\rho=\frac{1-r}{1-c}, \quad (4)$$

the active (A) state. Linearization around $\rho=0$ shows that the D state is stable for $r>1$ and unstable for $r<1$. Linearization around $\rho=1$ shows that the C state is stable for $c>r$ and unstable for $c<r$.

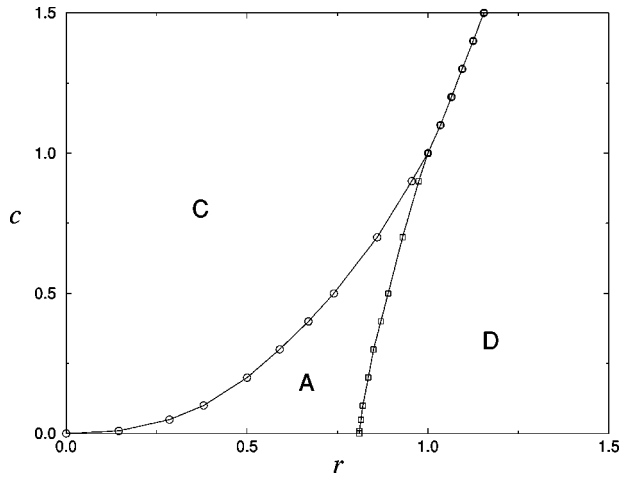

FIG. 1. Phase diagram in the (r,c) plane obtained from numerical simulation on a square lattice. The three phases are full cooperator (C) state for which $\rho=1$, active state (A) for which $0<\rho<1$, and full defector state (D) for which $\rho=0$. The transition lines C-A and A-D are critical lines whereas the line C-D is a discontinuous transition line. The three lines meet at the point $r=c=1$.

III. NUMERICAL SIMULATION

We have simulated the model in one and two dimensions. The model exhibits two frozen (absorbing) states: The full cooperator state (C) in which $\rho=1$ and the full defector (D) state in which $\rho=0$. In addition to these two frozen states the model may display an active state (A) for which $0<\rho<1$. Our simulations indicate that the active state is present in two dimensions but not in one dimension.

The phase diagram for the one-dimensional case displays only the two frozen states separated by a first order transition line. The full defection state occurs for $r>1$ whereas the full cooperation state occurs for $r<1$, independent of the value of c. The results were obtained for a chain of 1000 sites with periodic boundary conditions. The initial state was a complete random state, that is, a configuration in which each site was occupied by a cooperator or by a defector with equal probability. The number of Monte Carlo steps varied from 10^4 to 10^6.

In two dimensions, we have simulated the model on a square lattice with 100×100 sites starting with a complete random configuration of players. We have used a number of Monte Carlo steps that varied from 10^4 to 10^6. The phase diagram, shown in Fig. 1, displays three phases: the frozen full cooperation state (C) with $\rho=1$, the frozen full defection state (D) with $\rho=0$, and the active state (A) with $0<\rho<1$. The active state is present only within the region $r<1$ and $c<1$. In this region there are two transition lines corresponding to the transitions C-A and A-D. The density ρ varies continuously as one crosses these two lines. In the region $r>1$ and $c>1$ there is a transition line corresponding to the transition C-D in which the density jumps from the value $\rho=1$ to $\rho=0$. The three lines meet at the point $c=r=1$.

As one varies r for a fixed value of $c<1$ the density ρ varies continuously as shown in Fig. 2. Each point in this figure was calculated by using 10^4 Monte Carlo steps. The transition from the A state to the frozen D state is a continuous phase transition and its critical behavior is in the same universality class as the direct percolation model. We have

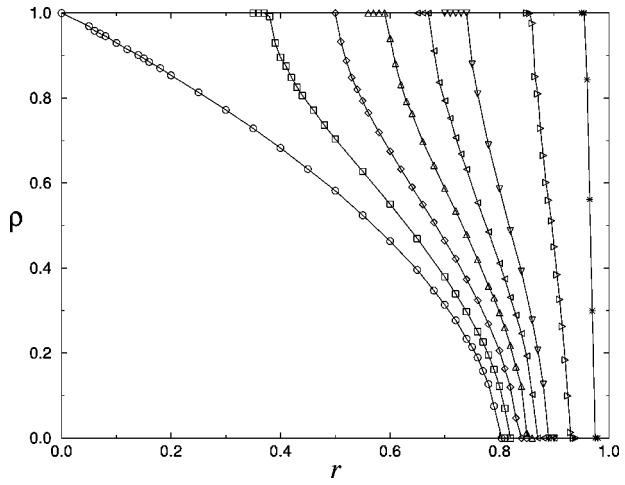

FIG. 2. Density of cooperator ρ versus the parameter r for several values of c, obtained from numerical simulations. From left to right the values of c are 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.7, and 0.9.

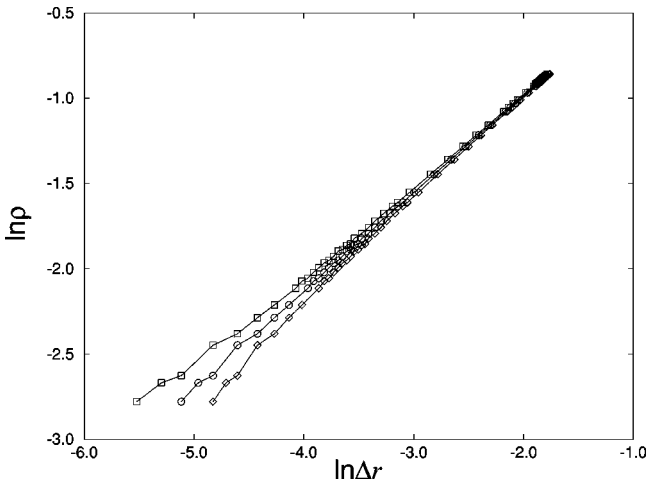

FIG. 3. Plot of $\ln\rho$ versus $\ln\Delta r$ where $\Delta r = r_c - r$ for the case $c=0$. The three lines correspond, from left to right, to tentative values of $r_c = 0.798$, 0.800, and 0.802. The best linear fit gives $r_c = 0.800 \pm 0.001$ and the critical exponent $\beta = 0.58 \pm 0.01$.

calculated the critical exponent β related to the order parameter and found results which are in fair agreement with the known results for such a universality class. As an example, we have plotted in Fig. 3, for the case of $c=0$, the $\ln\rho$ versus $\ln(r_c - r)$ and seek the best linear fit. We have found for this case $r_c = 0.800 \pm 0.001$ and, from the slope of this plot, the critical exponent $\beta = 0.58 \pm 0.01$, which is in fair agreement with results for direct percolation in $2+1$ dimensions [10]. Each point was calculated by using 10^6 Monte Carlo steps. Also, as expected, we found that, for $0 < c < 1$ the transition from the A state to the frozen C state is also continuous and belongs to the same universality class.

We remark that the case $r = c = 1$ corresponds to the voter model [11]. It is known that in two dimensions the voter model has only two stationary states independent of the initial conditions [11]. In the present case they correspond to the frozen C and D states.

Our final result comes from the comparison with a lattice with a few number of players interacting according to the same rules. In this case, one observes only the two frozen states. For fixed values of the transition rates one reaches either state depending on the initial conditions. As the number of players grows to a large number of players the model ends up exhibiting only one of them, or the active state, independent of the initial condition. So, this comparison helps to show clearly that cooperation or defection are collective phenomena that do not depend on how the players choose their actions in the beginning of the game. It is important to note that such a behavior is typical for all systems presenting absorbing states.

IV. CONCLUSIONS

The first result of the analysis conducted above is that the model, in two dimensions, exhibits, in a way similar to that of Szabó and Tőke [9], two frozen (absorbing) states: the full cooperation ($\rho = 1$) state and the full defection ($\rho = 0$) state. In addition, as a summary of the above, the model also displays an active state for which $0 < \rho < 1$. The full defection state occurs for sufficiently large values of r and the full cooperation state occurs for sufficiently large values of c. For certain combinations of c and r we also have an active state.

The second result has to do with the critical exponent associated to the power law behavior of the order parameter when the transition rates approach their critical values. Our numerical results for the critical exponent β put the present model into the universality class of direct percolation as expected according to the conjecture by Grassberger [12] and Janssen [13].

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